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HEAVY QUARK MASSES

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HEAVY QUARK MASSES*

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Abstract

A simple argument is given which identifies, in the large quark mass limit, the mass of the heavy-light pseudoscalar or scalar bound state, with the renormalized mass of the heavy quark.

Non perturbative numerical methods have been recently used to extract static properties of heavy hadrons⁽¹⁾. The difficulty in applying the usual lattice Monte Carlo method is that the heavy hadron has a mass much larger than the ultraviolet cutoff provided by the available lattices. A similar problem arises in the study of hadronic contributions to weak interactions, where the mass of the weak boson is over an order of magnitude larger than the realistic lattice spacing on which the hadrons propagate. In both cases the problem is solved, thank to asymptotic freedom, by integrating analytically the high energy degrees of freedom and then using lattice simulations to perform the remaining functional integration over a so-called effective theory in which all non perturbative features are expressed in terms of low energy degrees of freedom only.

In the case of heavy quark physics, this strategy has been applied to get the static properties of mesons composed of one heavy quark and one light antiquark in the infinite (renormalized) mass limit⁽²⁾. More recently, also the first order inverse mass corrections have been expressed through the effective low energy theory⁽³⁾. The mass parameter m_Q , in terms of which the $\frac{1}{m_Q}$ expansion is expressed, has been chosen conventionally to be the renormalized heavy quark mass, i.e. the mass the heavy quark would have if it were not confined. In this context, it is important to know exactly which value for m_Q is to be

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used when comparing with actual experimental data, i.e. m_Q is to be expressed in terms of some directly measurable quantity.

An obvious way to fix its precise value would be to use as an input some static property computed up to order $\frac{1}{m_Q}$, thus loosing one of the predictions of the theory. It is the purpose of this note to show that, up to $\frac{1}{m_Q}$ corrections, the value of m_Q must be precisely identified with mass of the pseudoscalar $Q\bar{q}$ bound state, usually denoted by B:

$$m_Q = m_B \quad (1)$$

It is remarkable that eq.(1) is exact, up to $\frac{1}{m_Q}$ corrections, at all orders of the strong coupling constant. The demonstration of eq.(1) is quite easy and starts from the partial conservation of the axial $\hat{Q}q$ current:

$$\begin{aligned} \partial_\mu A_\mu^{\hat{Q}q} &= m_Q^{(0)} \hat{Q} \gamma_5 q \\ A_\mu^{\hat{Q}q} &\equiv i \hat{Q} \gamma_5 \gamma_\mu q \end{aligned} \quad (2)$$

In eq.(2) we have set the light quark mass to zero, as it would only contribute to $\frac{1}{m_Q}$ corrections.

$m_Q^{(0)}$ denotes the bare mass of the heavy quark, so that eq.(2) is to be thought suitably regularized with a cutoff large enough not to disturb the heavy quark propagation. While the matrix elements of $A_\mu^{\hat{Q}q}$ are ultraviolet convergent, both $m_Q^{(0)}$ and $\hat{Q} \gamma_5 q$ are ultraviolet divergent, their product being finite.

In the static, large mass limit the operators $A_0^{\hat{Q}q}$ and $\hat{Q} \gamma_5 q$ are proportional, i.e.:

$$\frac{\langle \alpha | A_0^{\hat{Q}q} | \beta \rangle}{\langle \alpha | \hat{Q} \gamma_5 q | \beta \rangle} = \text{independent of } |\alpha\rangle \text{ and } |\beta\rangle \quad (3)$$

This property is essentially due to the fact that the matrix γ_0 can be shifted to operate on the heavy quark field, giving 1 up to computable perturbative corrections.

We can now sandwich eq.(2) between the vacuum and a B meson at rest:

$$\langle 0 | \partial_\mu A_\mu^{\hat{Q}q} | B \rangle = -i m_B \langle 0 | A_0^{\hat{Q}q} | B \rangle = m_Q^{(0)} \langle 0 | \hat{Q} \gamma_5 q | B \rangle \quad (4)$$

which gives:

$$m_B = i m_Q^{(0)} \frac{\langle 0 | \bar{Q} \gamma_5 q | B \rangle}{\langle 0 | A_0^Q q | B \rangle} \quad (5)$$

The ratio of matrix elements in the r.h.s. of eq.(5) is universal (i.e. independent of the external states) in virtue of eq.(3), although ultraviolet divergent.

If quarks were not confined, as is the case at every order in perturbation theory, we could take the matrix elements of eq.(2) between a heavy and a light quark:

$$\langle q | \partial^\mu A_\mu^Q q | Q \rangle = -i m_Q \langle q | A_0^Q q | Q \rangle = m_Q^{(0)} \langle q | \bar{Q} \gamma_5 q | Q \rangle \quad (6)$$

where m_Q denotes the renormalized heavy quark mass, i.e. the position of the pole in the heavy quark propagator. From eqs.(5), (6) and the universality property eq.(3), eq.(1) follows at once.

An identical argument, starting with the divergence of the vector current, leads to the result that, again to the leading order in $\frac{1}{m_Q}$, also the mass of the heavy-light quark scalar bound state is identical with the renormalized heavy quark mass.

In conclusion, our result, eq.(1), allows a parameter free comparison of experimental data with the lattice computations based on ref.(3).

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